

Rechnung:

$$\textcircled{1a} \cdot dS = \sqrt{(x')^2 + (y')^2 + (z')^2} dt = \sqrt{1^2 + t^2 + (at)^2} dt =$$

$$= \sqrt{1+17t^2} dt.$$

$$\bullet \sqrt{2y} = \sqrt{2 \cdot \frac{3}{2} t^2} = t.$$

$$\int \sqrt{2y} dS = \int_0^1 t \cdot \sqrt{1+17t^2} dt = \frac{1}{2} \cdot \frac{1}{17} \cdot \frac{2}{3} (1+17t^2)^{3/2} \Big|_0^1 = \frac{1}{51} (18^{3/2} - 1)$$

$$\textcircled{1b} \cdot y = \frac{2}{3} X^{3/2}; \quad y' = X^{1/2}; \quad dS = \sqrt{1+(y')^2} dx = \sqrt{1+x'} dx$$

$$\int_L \frac{y}{\sqrt{x}} dS = \int_3^8 \frac{\frac{2}{3} X^{3/2}}{\sqrt{x}} \sqrt{1+x'} dx = \frac{2}{3} \int_3^8 \widehat{x} \sqrt{1+x'} dx = \frac{2}{3} \left(x \cdot \frac{2}{3} (1+x')^{3/2} \Big|_3^8 - \right. \\ \left. - \frac{2}{3} \int_3^8 (1+x')^{3/2} dx \right) = \frac{2}{3} \left(\frac{2}{3} \left(8 \cdot 27 - 3 \cdot 8 \right) - \frac{2}{3} \cdot \frac{2}{5} (1+x')^{5/2} \Big|_3^8 \right) = \\ = \frac{4}{9} \cdot 8 \cdot 24 - \frac{4}{9} \cdot \frac{2}{5} (3^5 - 2^5) = \textcircled{...}$$

$$\textcircled{2a} \int (x^3 + 2\sqrt{y}) dx + 4\sqrt[3]{x^4 y} dy; \quad \begin{array}{l} y = x^2 \\ dy = 2x dx \end{array} \quad 0 \leq x \leq 1.$$

$$\int_0^1 (x^3 + 2\sqrt{x^2}) dx + 4 \int_0^1 \sqrt[3]{x^4} \cdot 2x dx = \int_0^1 (x^3 + 2x + 8x^3) dx =$$

$$= \left(\frac{9x^4}{4} + x^2 \right) \Big|_0^1 = \frac{9}{4} + 1 = \textcircled{\frac{13}{4}}$$

$$\textcircled{2b} \quad A = \iint_{(0,0)}^{\textcircled{1,2}} \vec{F} \cdot d\vec{S} = \iint_{(0,0)}^{\textcircled{1,2}} 2xy dx + x^2 dy; \quad \begin{array}{l} P_y = 2x \\ Q_x = 2x \end{array}, \quad \text{r. K.}$$

zu A kehren wir den Vektor rückwärts.

$$A = \int_0^1 2 \cdot 0 dx + \int_0^2 1^2 dy = \textcircled{2}.$$

(3a) $du = \frac{x}{\sqrt{x^2+y^2}} dx + \frac{y}{\sqrt{x^2+y^2}} dy ; u=?$

(1) Проверка: $P'_y = Q'_x : x \cdot \left(-\frac{1}{2}\right)(x^2+y^2)^{-3/2} \cdot 2y = y \left(-\frac{1}{2}\right)(x^2+y^2)^{-3/2} \cdot 2x -$
беспр.

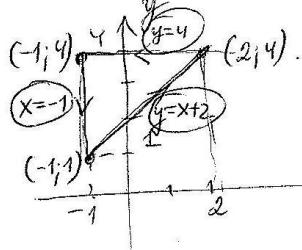
(2) $u = \int_{(1;0)}^{(x,y)} \frac{x dx}{\sqrt{x^2+y^2}} + \frac{y dy}{\sqrt{x^2+y^2}} + C$
 $u = \int_1^x \frac{1}{\sqrt{x^2+y^2}} dx + \int_0^y \frac{y dy}{\sqrt{x^2+y^2}} + C = x \left|_1^x + \frac{1}{2} \cdot 2\sqrt{x^2+y^2} \right|_0^y + C$
 $u = x - 1 + \sqrt{x^2+y^2} - x + C$
 Ответ: $\boxed{u = \sqrt{x^2+y^2} + C}$

(3b) $\vec{a} = (y + \ln(x+1)) \vec{i} + (x+1 - e^y) \vec{j} ; \text{ искомая } u=?$

(1) Проверка: $P'_y = Q'_x : 1 = 1 - \text{беспр.} \Rightarrow \text{поступательное}$.

(2) $u = \int_{(0;0)}^{(x,y)} (y + \ln(x+1)) dx + \int_0^y ((x+1) - e^y) dy + C$.
 $u = \int_0^x \ln(x+1) dx + \int_0^y ((x+1) - e^y) dy + C =$
 $= [(x+1)\ln(x+1) - x] \Big|_0^x + [(x+1)y - e^y] \Big|_0^y + C =$
 $= (x+1)\ln(x+1) - x + (x+1)y - e^y + 1 + C$.
 Ответ: $\boxed{u = (x+1)\ln(x+1) - x + (x+1)y - e^y + C^*}$.

$$(4) \oint_L xy \, dx + (x-2y) \, dy$$



(a) Koeffizienten:

$$\oint_L = \int_{L1} + \int_{L2} + \int_{L3}$$

$$L_1: \left| \begin{array}{l} y = x+2 \\ dy = dx \\ -1 \leq x \leq 2 \end{array} \right| ; \int_{L1}^{x+2} xy \, dx + (x-2y) \, dy = \int_{-1}^2 [x(x+2) + x - 2(x+2)] \, dx = \\ = \int_{-1}^2 (x^2 + 2x + x - 2x - 4) \, dx = \left(\frac{x^3}{3} + \frac{x^2}{2} - 4x \right) \Big|_{-1}^2 = \frac{9}{3} + \frac{3}{2} - 4 \cdot 3 = \\ = 3 + \frac{3}{2} - 12 = \frac{3}{2} - 9 = \boxed{-\frac{15}{2}}$$

$$L_2: \left| \begin{array}{l} y = 4 \\ dy = 0 \\ 2 \geq x \geq -1 \end{array} \right| ; \int_{L2}^4 xy \, dx + (x-2y) \, dy = \int_{-1}^2 4x \, dx = 2x^2 \Big|_{-1}^2 = 2 \cdot (-3) = \boxed{-6}$$

$$L_3: \left| \begin{array}{l} x = -1 \\ dx = 0 \\ 4 \geq y \geq 1 \end{array} \right| ; \int_{L3}^4 xy \, dx + (x-2y) \, dy = \int_1^4 (-1-2y) \, dy = \\ = \int_1^4 (1+2y) \, dy = \left(y + y^2 \right) \Big|_1^4 = 3 + 15 = \boxed{18}$$

$$\oint_L = -\frac{15}{2} + (-6) + 18 = -\frac{15}{2} + \cancel{12} = \boxed{\frac{9}{2}}$$

(5) no polynomne Funktion.

$$\oint_D P \, dx + Q \, dy = \iint_D (Q'_x - P'_y) \, dx \, dy = \iint_D (1-x) \, dx \, dy = \\ = \int_{-1}^2 (1-x) \, dx \int_0^4 dy = \int_{-1}^2 (1-x)(2-x) \, dx = \int_{-1}^2 (2-3x+x^2) \, dx = \\ = \left(2x - \frac{3x^2}{2} + \frac{x^3}{3} \right) \Big|_{-1}^2 = 2 \cdot 3 - \frac{3}{2} \cdot 3 + \frac{1}{3} \cdot 9 = 6 - \frac{9}{2} + 3 = \boxed{\frac{9}{2}}$$